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We propose a general scheme for entangling the spins (or any spin-like degree of freedom) of two particles of any type (bosons or fermions) by a combination of two particle interferometry and which way detection. We show how the fractional yield of entangled pairs per input pair can be arbitrarily increased. We show that the same setup allows identification of the quantum statistics of the incident particles through spin correlation measurements. Our setup also exhibits a curious complementarity between particle distinguishability and the amount of entanglement generated.

Recent years have witnessed a great surge of interest in the applications of entanglement in quantum communications [1–4]. In this context, it is important to explore efficient and general ways of preparing entanglement. Most known mechanisms for obtaining entangled states [4–8], are dependent on the specific nature of the systems involved. Here we propose a very general scheme for entangling the spins (or any spin-like degree of freedom) of two particles of any type (bosons or fermions) by a combination of two particle interferometry and which way detection. The fractional yield of entangled pairs per input pair can be arbitrarily increased by increasing the number of beam splitters in the setup. We show that the same setup allows identification of the quantum statistics of the incident particles through spin correlations as opposed to previous tests based on particle number measurements [9–13]. It also exhibits a complementarity between particle distinguishability and the amount of entanglement produced. This complementarity, involving "which particle" information in two particle interference, differs fundamentally from that involving "which way" information in single particle interference [14–17].

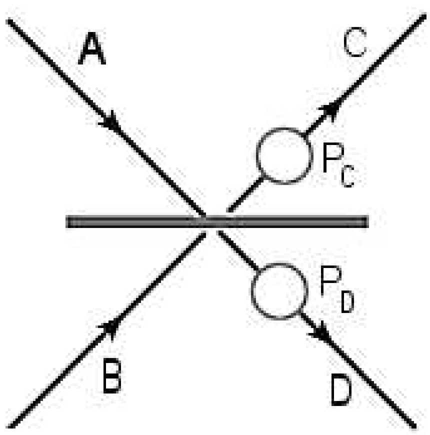

FIG. 1. A preliminary setup consisting of a beam splitter (input paths A and B, output paths C and D) and absorptionless path detectors P_C and P_D which do not disturb the spin. When a pair of identical particles with opposite spins are incident on the first beam splitter, one from arm A and the other from arm B, then corresponding coincidence in P_C and P_D, a spin entangled state is generated.

We begin by describing a preliminary setup which produces entangled states with 50 percent efficiency (successful cases being identifiable by certain detector clicks). Fig.1 depicts the setup composed of a beam splitter with input channels A and B, output channels C and D and which-channel detectors P_C in C and P_D in D. These detectors are assumed to be *nonabsorbing* and are able to *determine the path without disturbing the spin* (this is possible since position and spin commute; feasibility discussed later). Consider two identical particles in different spin states (say $|\uparrow\rangle$ and $|\downarrow\rangle$) incident simultaneously on the beam splitter from arms A and B as shown in Fig.1. This state, in second quantized notation, is described as $a^\dagger_{A\uparrow}a^\dagger_{B\downarrow}|0\rangle$ where $|0\rangle$ is the vacuum state and $a^\dagger_{A\uparrow}$ and $a^\dagger_{B\downarrow}$ are creation operators for $\uparrow$ spin in path A and $\downarrow$ spin in path B respectively. We will label the state concisely as $|A\uparrow;B\downarrow\rangle$. For fermions, $|A\uparrow;B\downarrow\rangle = -|B\downarrow;A\uparrow\rangle$ and for bosons $|A\uparrow;B\downarrow\rangle = |B\downarrow;A\uparrow\rangle$. The transformation done by the beam splitter is [13,18]

$$|A\uparrow;B\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}\{\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)\} + \frac{i}{2}(|C\uparrow;C\downarrow\rangle + |D\uparrow;D\downarrow\rangle), \tag{1}$$

where the $+$ sign stands for fermions and the $-$ sign stands for bosons.

After detector clicks, the combined state of the particles and the detectors is

$$\frac{1}{\sqrt{2}}\{\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)\}|P_C^*\rangle|P_D^*\rangle + \frac{i}{2}(|C\uparrow;C\downarrow\rangle|P_C^*\rangle|P_D\rangle + |D\uparrow;D\downarrow\rangle|P_C\rangle|P_D^*\rangle), \tag{2}$$

where $|P_C\rangle$ and $|P_D\rangle$ are the unexcited and $|P_C^*\rangle$ and $|P_D^*\rangle$ are the excited (corresponding to detection of one or more particles) detector states. When the detectors are found in the state $|P_C^*\rangle|P_D^*\rangle$ (coincidence), the state of the particles is projected onto the spin entangled state $\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)$. The spin part of this state can be rewritten in the first quantized notation (using the paths as particle labels) as $\frac{1}{\sqrt{2}}(|\uparrow\rangle_D|\downarrow\rangle_C \pm |\downarrow\rangle_D|\uparrow\rangle_C)$.

It is fully legitimate to use the paths as particle labels because the particles are *identical* (the same labeling is used for photon pairs exiting a parametric down converter [5]).

We should mention that the method of generating entanglement described above has been closely anticipated (for bosons) in earlier literature involving two photon interferometry [19,20]. However, in these schemes, path and polarization (spin) were measured simultaneously, and after the measurements, the resource of entanglement was *not available* for applications. It is the special type of detectors in our scheme that help us to obtain an useful *source* of spin entangled particles.

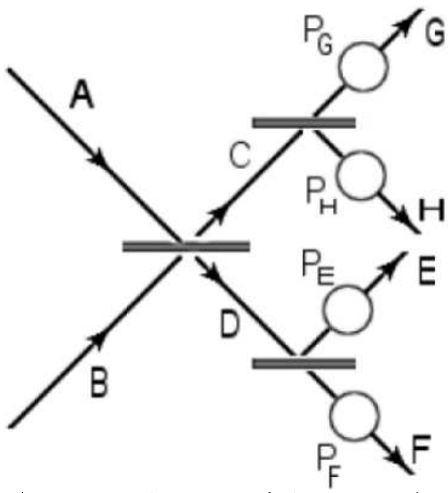

FIG. 2. An improved version of the setup. A and B are the input paths, E,F,G and H are the output paths and P_E, P_F, P_G and P_H are absorptionless path detectors which do not disturb the spin. A pair of identical particles with opposite spins are incident on the first beam splitter, one from arm A and the other from arm B. For coincidence between any pair of detectors, which happens in 75 percent of the cases, a spin entangled state is emitted along the corresponding pair of paths

We now show that the efficiency of our scheme can be arbitrarily increased. First consider the addition of two more beam splitters to the setup as shown in Fig.2 with the four exit paths E, F, G and H being equipped with which-path detectors P_E, P_F, P_G and P_H respectively. If the state $|A\uparrow; B\downarrow\rangle$ is incident on the first beam splitter, then the final combined state of the particles and the detectors is

$$\begin{aligned}
\frac{1}{2\sqrt{2}}\{ &-\frac{1}{\sqrt{2}}(|G\uparrow;E\downarrow\rangle \pm |G\downarrow;E\uparrow\rangle)|P_E^*\rangle|P_F\rangle|P_G^*\rangle|P_H\rangle \\
&+\frac{i}{\sqrt{2}}(|H\uparrow;E\downarrow\rangle \pm |H\downarrow;E\uparrow\rangle)|P_E^*\rangle|P_F\rangle|P_G\rangle|P_H^*\rangle \\
&+\frac{i}{\sqrt{2}}(|G\uparrow;F\downarrow\rangle \pm |G\downarrow;F\uparrow\rangle)|P_E\rangle|P_F^*\rangle|P_G^*\rangle|P_H\rangle \\
&+\frac{1}{\sqrt{2}}(|H\uparrow;F\downarrow\rangle \pm |H\downarrow;F\uparrow\rangle)|P_E\rangle|P_F^*\rangle|P_G\rangle|P_H^*\rangle \\
&-\frac{1}{\sqrt{2}}(|F\uparrow;E\downarrow\rangle \mp |F\downarrow;E\uparrow\rangle)|P_E^*\rangle|P_F^*\rangle|P_G\rangle|P_H\rangle \\
&-\frac{1}{\sqrt{2}}(|H\uparrow;G\downarrow\rangle \mp |H\downarrow;G\uparrow\rangle)|P_E\rangle|P_F\rangle|P_G^*\rangle|P_H^*\rangle\} \\
&-\frac{i}{4}|E\uparrow;E\downarrow\rangle|P_E^*\rangle|P_F\rangle|P_G\rangle|P_H\rangle \\
&-\frac{i}{4}|G\uparrow;G\downarrow\rangle|P_E\rangle|P_F\rangle|P_G^*\rangle|P_H\rangle \\
&+\frac{i}{4}|F\uparrow;F\downarrow\rangle|P_E\rangle|P_F^*\rangle|P_G\rangle|P_H\rangle \\
&+\frac{i}{4}|H\uparrow;H\downarrow\rangle|P_E\rangle|P_F\rangle|P_G\rangle|P_H^*\rangle, \qquad (3)
\end{aligned}$$

where the upper sign stands for fermions and the lower sign for bosons. The above expression indicates that there will be coincidence between a pair of detectors in 75 percent of the cases. In these cases, a spin entangled state will be generated along the corresponding pair of exit channels. For example, for fermions, if P_E and P_G click, the triplet state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_G|\downarrow\rangle_E + |\downarrow\rangle_G|\uparrow\rangle_E)$ is produced and if P_H and P_G click, the singlet state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_H|\downarrow\rangle_G - |\downarrow\rangle_H|\uparrow\rangle_G)$ is produced. Based on the knowledge of the detector clicks, all the different entangled states can be converted to a desired entangled state by applying spin dependent phases along appropriate paths. Only in 25 percent of the cases detector clicks will result in a disentangled state.

The above improvement in success probability stems from the fact that the extra pair of beam splitters not only map the entangled part $\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)$ of the previous state (Eq.(1)) to entangled final parts, but also map 50 percent of the disentangled parts $|C\uparrow;C\downarrow\rangle$ and $|D\uparrow;D\downarrow\rangle$ to entangled final parts. We can easily double the number of output channels by subdividing each of the existing outputs by beam splitters and at each stage the entangled fraction increases. If we subdivide in this manner to obtain 2^N outputs, and put detectors only in these final exit paths, the fractional yield of entangled pairs is $1-1/2^N$. For $N=7$, this exceeds 99 percent.

The next issue of the paper is the identification of quantum statistics through spin correlation measurements. Consider Fig.1 once again and the incident state $|A\uparrow; B\downarrow\rangle$. If there is a detector coincidence, then the unitary operation

$$\begin{aligned}
|\uparrow\rangle &\to \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \\
|\downarrow\rangle &\to \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \qquad (4)
\end{aligned}$$

is applied to the spins of particles in each of the output channels and then the spins of the particles are measured in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis. There will be *perfect correlation* between the spin measurement outcomes in the two paths for *fermions* and *perfect anticorrelation* for *bosons*. The signature of the spin correlation can thus be used to identify quantum statistics of the incident particles.

This method differs from all earlier schemes which rely on particle number measurements for identifying statistics [9–13].

We now describe a curious complementarity between particle distinguishability and entanglement in our scheme. Complementarity of "which path" information with fringe contrast in single particle interference has attracted great interest [14–17]. In two particle interferometry, "which path" information is naturally replaced by "which particle" information. The particles impinging on our setup (Fig.1) are indistinguishable apart from their spins (which we choose not to measure as we intend to create a spin entangled state). Suppose the particles were partially or fully distinguishable through some other observable such as energy or momentum or any non-spin internal degree of freedom. For example, suppose, the incident state was $|A\uparrow S_1; B\downarrow S_2\rangle$, with $|\langle S_1|S_2\rangle| = a \leq 1$. Then the state of the two particles created due to detector coincidence is $\frac{1}{\sqrt{2}}(|D\uparrow S_1; C\downarrow S_2\rangle \pm |D\downarrow S_2; C\uparrow S_1\rangle)$. The spin state of the particles (in first quantized notation) is

$$\rho = \frac{1}{2}\ (\ |\uparrow_C\downarrow_D\rangle\langle\uparrow_C\downarrow_D| + |\downarrow_C\uparrow_D\rangle\langle\downarrow_C\uparrow_D| \\ \pm |a|^2|\uparrow_C\downarrow_D\rangle\langle\downarrow_C\uparrow_D| \pm |a|^2|\downarrow_C\uparrow_D\rangle\langle\uparrow_C\downarrow_D|) \quad (5)$$

For the above state, a certain entanglement measure called concurrence [21] is $\mathbf{E} = |a|^2$. The probability of successful discrimination between states $|S_1\rangle$ and $|S_2\rangle$ (which is a measure of particle distinguishability) is $\mathbf{D} = 1 - |a|^2$. Thus we have, in analogy with Englert's relation in single particle interference [17], the following testable complementarity relation

$$\mathbf{E} + \mathbf{D} = 1. \quad (6)$$

The concurrence $\mathbf{E}$ for ρ can be inferred by measuring the expectation value of the Bell-CHSH operator $\hat{a}\hat{b} + \hat{a}\hat{b'} + \hat{a'}\hat{b} - \hat{a'}\hat{b'}$ on the two particles (labeled by their paths C and D) with $\hat{a} = \sigma_x^C, \hat{a'} = \sigma_y^C, \hat{b} = \frac{1}{\sqrt{2}}(\sigma_x^D + \sigma_y^D), \hat{b'} = \frac{1}{\sqrt{2}}(\sigma_x^D - \sigma_y^D)$ and dividing the result by $\pm 2\sqrt{2}$.

Finally we briefly discuss the realizability of our scheme. Beam splitters are available for photons, electrons [22], neutrons [23] , atoms [15,18] and even macromolecules [24] and two particle interferometry is feasible with photons [10,11] and nearly feasible with electrons [22] and atoms [18]. A theoretical model of an absorptionless path detector that keeps spin unaffected, has been considered in the context of quantum state reduction [25]. Such detectors, are already available for electrons (based on effects of electric fields) [16], have been suggested for photons (based on crossed phase modulation) [26], and neutrons (based on momentum transfer) [27]. For atoms, consider one with hyperfine ground levels $|g_1\rangle, |g_2\rangle, |g_3\rangle$ and $|g_4\rangle$ which can be made to interact with a cavity field in Fock state $|n\rangle$ to undergo transitions $|g_1\rangle|n\rangle \rightarrow |g_2\rangle|n+1\rangle$ and $|g_3\rangle|n\rangle \rightarrow |g_4\rangle|n+1\rangle$ [28]. Then with an incident state $|Ag_1; Bg_3\rangle$ and by inducing the above transitions in cavitites placed in paths C and D, we will obtain the entangled state $\frac{1}{\sqrt{2}}(|g_2\rangle_D|g_4\rangle_C \pm |g_4\rangle_D|g_2\rangle_C)$ when both cavities are found in the state $|n+1\rangle$. For macromolecules, one can choose any two independent degrees of freedom, one for entangling and the other for path detection.

To summarize, we have presented an *efficient* scheme for entangling two particles of *any type* (bosons or fermions). This is important, as entangled states of particles such as neutrons, electrons or macromolecules are yet to be prepared. Our scheme provides strong motivation for developing two particle interferometry in various systems in tandem with absorptionless "which-way" detectors. That the *same* setup can be used to test quantum statistics through spin correlations, and probe complementarity in two particle interference, is a significant feature. From a theoretical perspective, our work points towards potential connections between entanglement, quantum statistics and complementarity, which calls for further study.

We thank D. Atkinson and D. Bouwmeester for some useful discussions. DH is grateful for the support provided by the Royal Society - Indian National Science Academy Exchange Programme which helped to initiate this collaboration. We also acknowledge the First SERC Winter School on Foundations of Quantum Mechanics and Quantum Optics (Department of Science and Technology, Govt. of India).

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